UK Maths Trust

Intermediate Mathematical Olympiad

Hamilton paper

Thursday 21 March 2024

supported by G-Research Overleaf

England & Wales: Year 10 | Scotland: S3 | Northern Ireland: Year 11

These problems are meant to be challenging.

Try to finish whole questions even if you cannot do many; you will have done well if you hand in a complete solution to two or more questions.

Instructions

1. Time allowed: **2 hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**.
4. **Each question carries 10 marks**.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until **8am GMT on Saturday 23 March**.
11. Do not turn over until told to do so.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

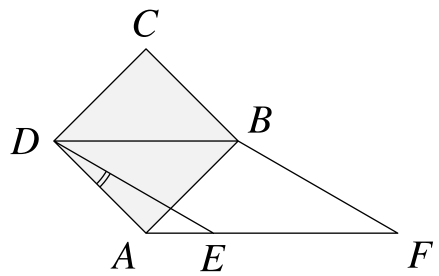
www.ukmt.org.uk

Advice to candidates

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but* not *decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do* not *hand in rough work.*

1. Richard is cycling at a speed v km/h when he looks at his cycle computer to see how long it will take him to get home at his current speed. It shows a time t hours.
He cycles at this speed for 40 minutes, then instantaneously slows down by 1 km/h and checks his cycle computer; the predicted time to get home at his new speed is still t hours.
After cycling at this new speed for 45 minutes, he instantaneously slows down by another 1 km/h and checks his cycle computer; the predicted time to get home at this speed is again t hours.
How far from home was he when he first looked at the computer?

2. $ABCD$ is a square. $BDEF$ is a rhombus with A, E and F collinear. Find $\angle ADE$.



3. A large number of people arrange themselves into groups of 2, 5 or 11 people. The mean size of a group is 4. However, when each person is asked how many other people are in their group (excluding themselves), the mean of their answers is 6. Prove that the number of groups must be a multiple of 27.

4. The numbers 1, 2, 3, 4 and 5 are used once each in some order substituting for the letters in the series of powers $M^{\left(A^{\left(T^{\left(H^{S}\right)}\right)}\right)}$. In how many of the arrangements is the units digit of the value of this expression equal to 1?

5. The integers 1 to 100 are written on a board. Seth chooses two distinct integers from the board, b and c, and forms the quadratic equation $x^2 + bx + c = 0$. If the quadratic equation formed has integer solutions, then he erases b and c from the board; if not, the board remains unchanged.
If Seth continually repeats this process, is it possible for him to erase all the numbers from the board?

6. The diagram shows a quadrilateral $ABCD$ with $AB + CD = BC$. The interior angle bisectors of $\angle B$ and $\angle C$, and the perpendicular bisector of AD, are shown as dotted lines. Prove that those three bisectors meet at a point.